

Algebra 2 Honors  
Final Exam 2015 – 2016 REVIEW

Below are examples of problems I might ask on a final exam, organized by function. At the end of the guide are practice problems to find the solutions to systems of equations.

**PLEASE NOTE:** The questions on your exam will ask you to relate the functions to each other, so make sure that you understand the relationships between them.

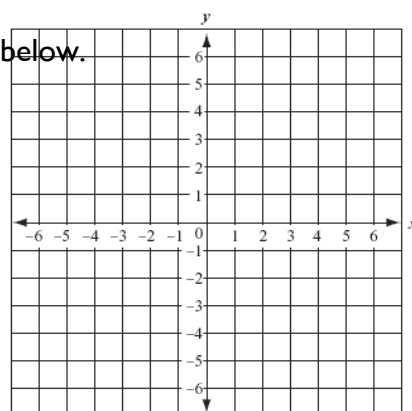
**EXPONENTIAL FUNCTIONS:**

- 1) Rationalize the denominator.  $\frac{10}{\sqrt[3]{5x^2}}$
- 2) Simplify  $32^{\frac{4}{5}}$
- 3)  $-\sqrt{2x-7} = 14$
- 4) Consider the equation  $\frac{9^{x^2}}{3^x} = 3$ . Select **all** that apply.
  - a. -3
  - b. -1
  - c.  $-\frac{1}{2}$
  - d. 1
  - e. 3
- 5) Simplify the expression. Remember, no negative exponents should be remaining.
  - a.  $\left(x^7 y^{-8} z^{\frac{1}{2}}\right)^4$
  - b.  $\left(\frac{a^m b^{-3}}{c^{-6} d}\right)^{10}$
  - c.  $(y^4 \cdot y^k)^m$
  - d.  $\left(\frac{z^2}{z^{-7}}\right)^{-5}$
  - e.  $\left(\frac{7^8}{7^x}\right)^{-2}$
  - f.  $\left(\frac{5^y}{13^x}\right)^{-6}$
- 6) Without graphing, identify if the following functions represent exponential **growth** or **decay**. **EXPLAIN** how you know.
  - a.  $f(x) = 0.4^x$
  - b.  $f(x) = 6.7^x$
  - c.  $f(x) = \left(\frac{3}{2}\right)^x$

Algebra 2 Honors  
Final Exam 2015 – 2016 REVIEW

- 7) Complete the table for  $f(x) = \left(\frac{1}{2}\right)^{x+1} - 3$ . Then sketch the function below.

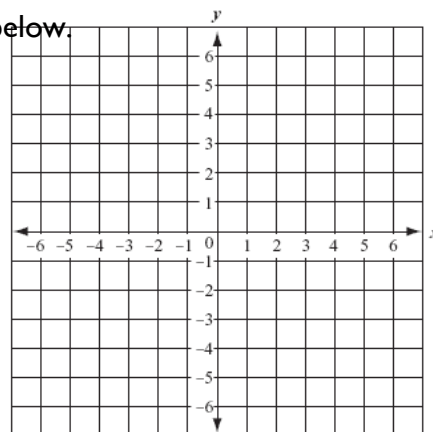
$x$	$f(x)$
-2	
-1	
0	
1	
2	



What is the domain and range of the function  $f(x) = \left(\frac{1}{2}\right)^{x+1} - 3$ ?

- 8) Complete the table for  $f(x) = (3)^{x-2} + 1$ . Then sketch the function below.

$x$	$f(x)$
-2	
-1	
0	
1	
2	



- 9) What is the domain and range of the function  $f(x) = (3)^{x-2} + 1$ ?

- 10) Compare the graph of the parent function,  $f(x) = 2^x$ , to the graph of  $g(x) = 2^{x+1} - 6$ . Describe how the graph has shifted.

- 11) Compare the graph of the parent function,  $f(x) = \left(\frac{1}{2}\right)^x$ , to the graph of  $g(x) = \left(\frac{1}{2}\right)^{x-7} + 2$ . Describe how the graph has shifted.

- 12) Rewrite the equation  $f(x) = 5(2)^{x+1} - 3$  in a simpler form so that it is represented by only two transformations of  $y = 2^x$ .

- 13) Rewrite the equation  $f(x) = 50(5)^{x-2} - 3$  in a simpler form so that it is represented by only two transformations of  $y = 5^x$ .

Algebra 2 Honors  
Final Exam 2015 – 2016 REVIEW

- 14) A scientist places 7.35 grams of a radioactive element in a dish. The half-life of the element is 2 days. After  $d$  days, the number of grams of the element remaining in the dish is given by the function  $R(d) = 7.35\left(\frac{1}{2}\right)^{d/2}$ . Which statement is true about the equation when it is rewritten without a fractional exponent? Choose **all** that apply.
- a) An approximately equivalent equation is  $R(d) = 7.35(0.250)^d$ .
  - b) An approximately equivalent equation is  $R(d) = 7.35(0.707)^d$ .
  - c) The base of the exponent in this form of the equation can be interpreted to mean that the element decays by 0.250 grams per day.
  - d) The base of the exponent in this form of the equation can be interpreted to mean that the element decays by 0.707 grams per day.
  - e) The base of the exponent in this form of the equation can be interpreted to mean that about 25% of the element remains from one day to the next day.
  - f) The base of the exponent in this form of the equation can be interpreted to mean that about 70.7% of the element remains from one day to the next day.
- 15)

**LOGARITHMIC FUNCTIONS:**

- 1) Write in log form:

$$2^5 = 32$$

$$3^7 = 2187$$

- 2) Write in exponential form:

$$\log_7 2401 = 4$$

$$\log 1000 = 3$$

- 3) Simplify:

$$\log_2 2^{3x}$$

$$\log^{3x} + \log^{2x}$$

- 4) Evaluate using a calculator. Round to 3 decimal places.

$$\log_{150} 12$$

$$\log_3 513$$

- 5) Expand by using the properties of logs to rewrite each expression as the sum or difference:

a)  $\log_4 \frac{\sqrt{5x}}{y^3}$

b)  $\log 5x^7y^8$

Algebra 2 Honors  
Final Exam 2015 – 2016 REVIEW

6) Condense the following expressions:

a)  $7\log_5 b - \frac{1}{3}\log_5 a$

b)  $7\log x + \frac{1}{2}\log y + \log(z - 7)$

7) Solve for x. Round to the nearest hundredth if necessary.

a)  $\log_5(x + 10) = \log_5 17$

d)  $6(4^{2x} - 7) = 1200$

b)  $(7^{2x} + 3)^2 = 58$

e)  $7\log_4 5x = 19$

c)  $8^{5x-3} + 7000 = 2090152$

f)  $\log_5 x - 5 = 120$

**QUADRATIC FUNCTIONS:**

1)  $2x^2 - 6x + 5 = 0$

2)  $4x^2 - 2x - 4 = 2$

3) The expression  $x^4 - 64$  can be rewritten in the following form where a, b, and c are real numbers:  $(x^2 + a)(x + b)(x + c)$ . Find the values of a, b, and c.

4)  $x^2 - 8x + 21 = (x - 4)^2 + 3x - 16$

5) The expression  $x^2(x - y)^3 - y^2(x - y)^3$  can be written in the form  $(x - y)^a(x + y)$ , where a is a constant. What is the value of a?

6) Consider the function  $f(x) = (2x - 1)(x + 4)(x - 2)$ . For what values of x is  $f(x) > 0$ ?

7) Consider a quadratic equation with integer coefficients and two distinct zeroes. If one zero is irrational, which statement is true about the other zero?

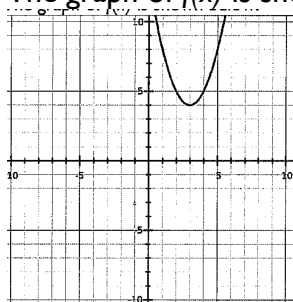
- a. The other zero must be rational.
- b. The other zero must be irrational.
- c. The other zero can be either rational or irrational.
- d. The other zero must be non-real.

8) A softball was thrown from 1<sup>st</sup> to 2<sup>nd</sup> base. The height, in feet, of the ball above the ground t seconds after being thrown can be determined by the expression  $-16t^2 + 40t + 4$ . What is the meaning of the 4 in the expression?

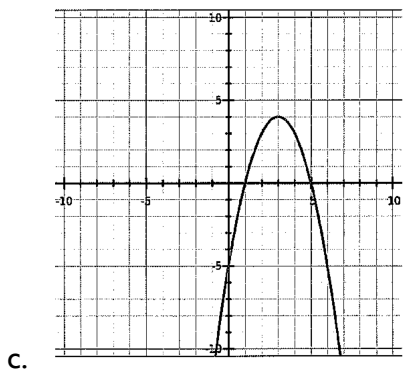
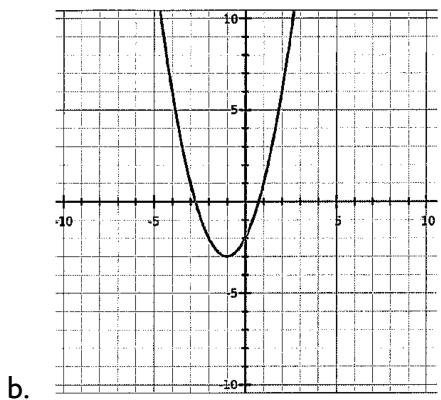
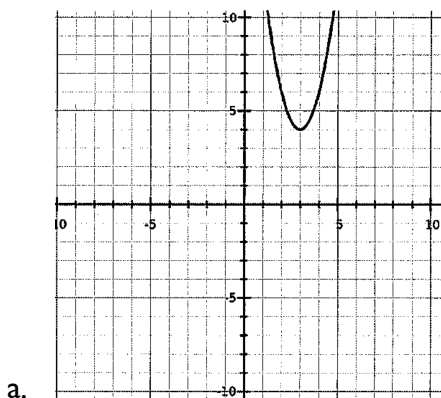
- a. The ball took 4 seconds to reach the ground.
- b. The ball took 4 seconds to reach its maximum height.
- c. The ball was thrown from a height of 4 feet.
- d. The ball reached a maximum height of 4 feet.

Algebra 2 Honors  
Final Exam 2015 – 2016 REVIEW

9) The graph of  $f(x)$  is shown below.



Identify the equations in the form  $y = k(x - r) + n$  which generate each of the graphs  $a - c$  as transformations of  $f(x)$ .



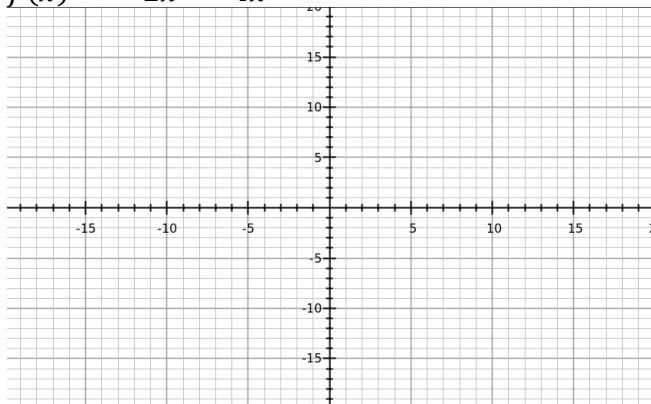
Algebra 2 Honors  
Final Exam 2015 – 2016 REVIEW

10) If  $a = 2$  and  $b = 5$ , determine what condition(s) on  $c$  will restrict the solutions for  $x$  to real numbers.

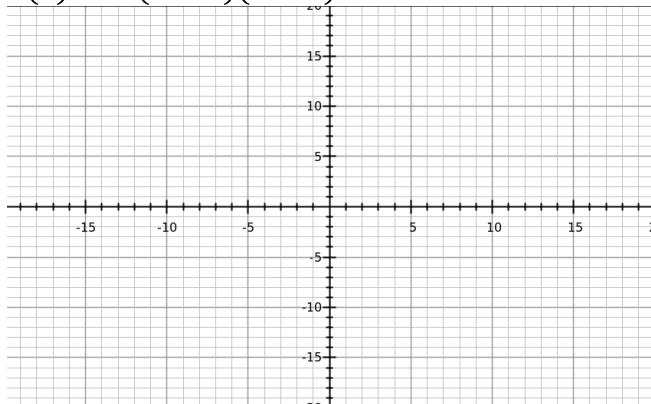
11) Graph and find each of these characteristics for each parabola.

direction      root(s)      y-intercept      vertex

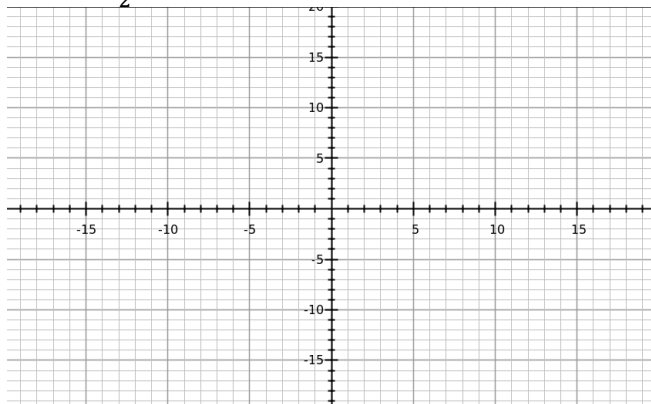
a.  $f(x) = -2x^2 - 4x$



b.  $h(x) = 2(x - 2)(x + 1)$

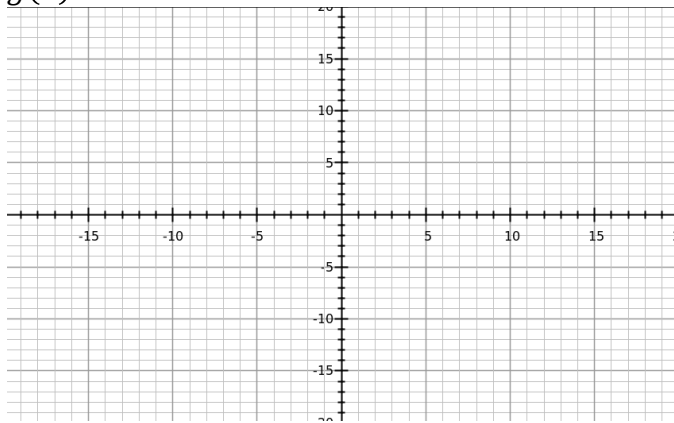


c.  $p(x) = \frac{1}{2}(x - 1)^2 + 4$



Algebra 2 Honors  
Final Exam 2015 – 2016 REVIEW

d.  $g(x) = 3x^2 - 2x + 2$

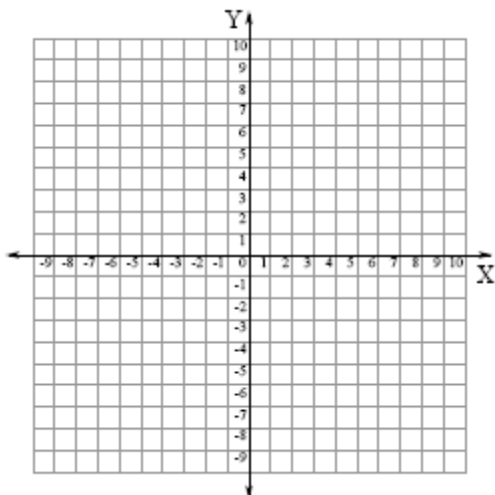


- 12) You jump off a 9-meter diving board. At the same time, your friend jumps off a 10-meter diving board next to you. However, you and your friend have different styles of jumping. You jump up off the board with an initial speed of 1.6m/sec. Your friend just steps off the board, without jumping at all. The functions describing your heights off the surface of the water in terms of time (in seconds) are

You:  $a(t) = -5t^2 + 1.6t + 9$

Friend:  $b(t) = -5t^2 + 10$

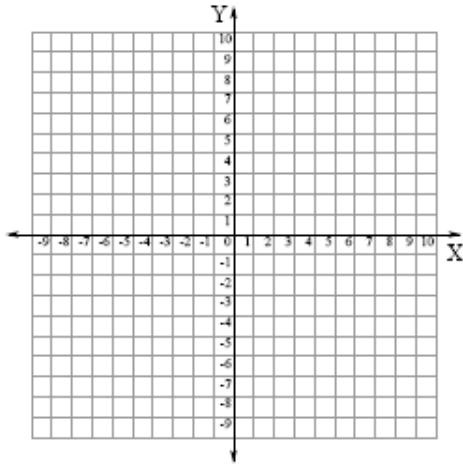
- Who hits the water first?
  - When you jump in the air, do you ever get as high as the 10-meter board? If not, how high do you get?
- 13) Write the equation of a parabola in vertex form, standard form, and factored form (if possible) that has a maximum value of 8 AND a y-intercept at (0, 6).
- 14) Graph the parabola of  $y = x^2 - 5x + 6$ . (2pts)



Algebra 2 Honors  
Final Exam 2015 – 2016 REVIEW

15) Write the equation from problem #4 in vertex form. (1pt)

16) Graph the parabola of  $y = 3(x - 6)^2 + 1$ . (2pts).



17) Write the equation from problem #6 in standard form (1pt.)

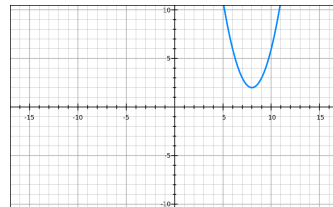
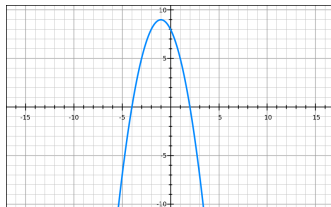
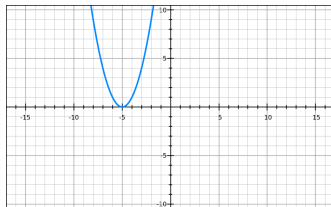
**Solve each equation. Leave all square roots in simplest radical form.**

18)  $7x^2 = 252$

19)  $2x^2 + 8x - 20 = 5x$

20)  $2x^2 + 4x + 3 = 0$

21) Choose one graph below. Describe its discriminant as positive, negative, or zero, and tell how you know this.



22) Which expression shows a correct use of the quadratic formula to solve the equation  $2x^2 = 5x - 1$ ?

- a.  $\frac{-5 \pm \sqrt{25-8}}{4}$
- b.  $\frac{-5 \pm \sqrt{25+8}}{4}$
- c.  $\frac{5 \pm \sqrt{25-8}}{4}$
- d.  $\frac{5 \pm \sqrt{25+8}}{4}$



Algebra 2 Honors  
Final Exam 2015 – 2016 REVIEW

23) What are the complex solutions to the following equation?

$$\frac{1}{2}x^2 - \frac{1}{2}x + \frac{1}{2} = 0$$

- a.  $5 \pm \frac{\sqrt{3}}{2}$
- b.  $5 \pm \frac{\sqrt{3}i}{2}$
- c.  $\frac{1 \pm i\sqrt{6}}{2}$
- d.  $\frac{1 \pm \sqrt{3}}{2}$

24) A polynomial is factored completely over the complex numbers.

Two of the factors are  $(x - 2i)$  and  $(x + 2i)$ . Which expression could be the one that was factored?

- a.  $x^4 - 3x^2 - 4$
- b.  $x^4 - 5x^2 + 4$
- c.  $x^4 - 5x^2 - 36$
- d.  $x^4 - 13x^2 + 36$

25)

**RATIONAL FUNCTIONS:**

1) Graph  $f(x) = \frac{2x-5}{x+4}$

2) Simplify. Make sure to state any restrictions on the variables.

$$\frac{9-x^2}{x^2+6x+9} \cdot \frac{3x+9}{3x-9}$$

3) Simplify. Make sure to state any restrictions on the variable.

$$\frac{4}{x^2-16} + \frac{3}{x^2+8x+16}$$

4) For a seesaw to be balanced,  $m_1d_1 = m_2d_2$ . Use the information in the table to write the function  $f(x)$  that allows you to determine  $m_1$ , the mass of the first child.

$m_2$	$d_1$	$d_2$
40	$10 - x$	$x$

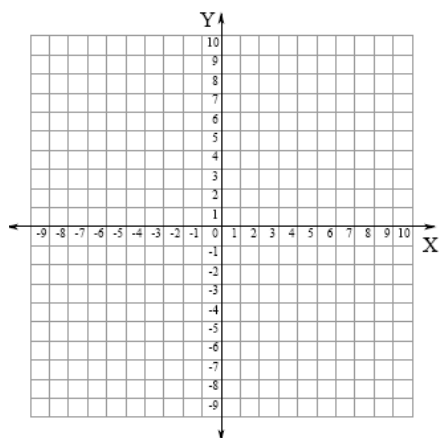
5) What points are on the graph of the equation  $\frac{x^2-8x+16}{x^2-2x-8}$ .

- a. (0, 16)
- b. (4, 0)
- c. (0, -2)
- d. (-2, 0)

Algebra 2 Honors  
Final Exam 2015 – 2016 REVIEW

- 6) A SmarTrip card on the Metro costs \$5 for the card and then \$2.10 for every Metro trip (assume for this problem that all trips cost the same amount). Write a function to show the average cost  $c$  of the SmarTrip card for  $t$  trips.
- 7) Identify the asymptotes of your function in the previous problem.
- 8) Write the four approach statements for your function in problem #1.

9) Graph  $y = \frac{5}{x-4} + 3$ .

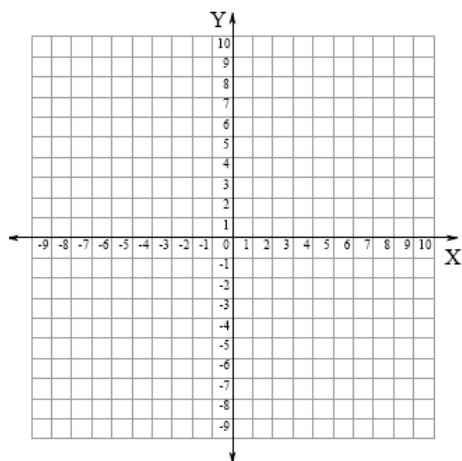


10) Rewrite  $y = \frac{5}{x-4} + 3$  to have asymptotes at  $x = -7$  and  $y = 6$ .

11) Identify the vertical asymptotes, horizontal asymptotes, and holes of the graph of  $\frac{2x^2 - 3x - 20}{x^2 - 6x + 8}$

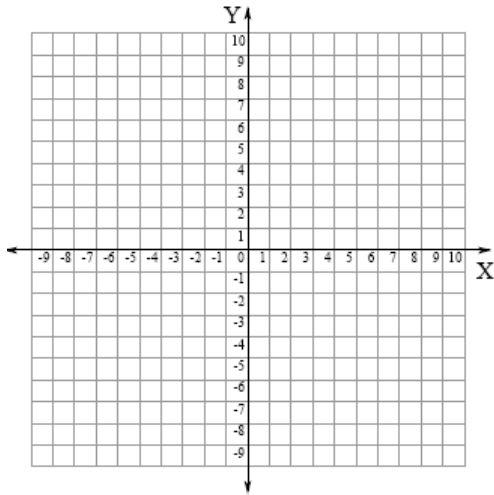
- Vertical asymptotes:
- Horizontal asymptotes:
- Holes:

12) Simplify and graph. Make sure to show all asymptotes and holes.  $\frac{x^2 + 4x - 12}{x + 6}$



Algebra 2 Honors  
Final Exam 2015 – 2016 REVIEW

- 13) Simplify and graph. Make sure to show all asymptotes and holes.  $\frac{5x-4}{x+3}$



- 14) Divide. Simplify your answer.

$$\frac{x^2 - 25}{x^2 - 16} \div \frac{2x + 10}{x^2 - 4x}$$

- 15) Find the LCM of  $2x + 4$  and  $x^2 + 2x$

16) Simplify  $\frac{7}{5x+25} - \frac{4}{3x+15}$

17) Simplify  $3x - \frac{x^2 + 5x}{x^2 - 2}$

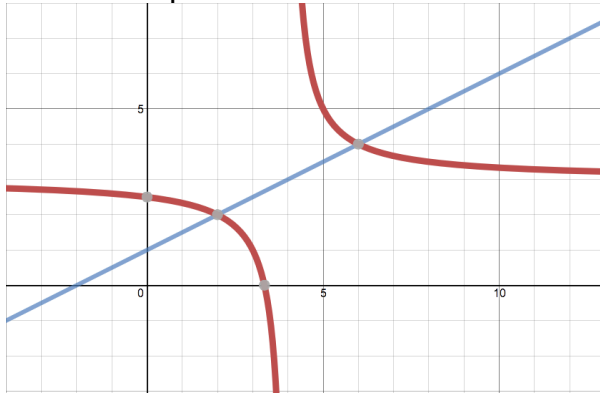
18) Simplify  $\frac{\frac{3}{2y}}{\frac{6}{8x}}$ .

19) Solve  $\frac{1}{4x} - \frac{3}{4} = \frac{7}{x}$

20) Solve  $\frac{2}{y} + \frac{1}{2} = \frac{5}{2y}$

Algebra 2 Honors  
Final Exam 2015 – 2016 REVIEW

21) Write the equation of each of the functions shown below.



22) Find the points of intersection of the two functions shown above. You must show algebraically how you arrived at your solution in order to earn credit.

23)

**FUNCTION TRANSLATIONS:**

1) The function  $f$  is defined as  $f(x) = \frac{1}{4}(2)^x$ . Write an expression that defines  $f(x + 3)$ .

2) Given these functions:

$$\text{ADD}(a, b) = a + b$$

$$\text{SUBTRACT}(a, b) = a - b$$

a) Find  $\text{ADD}(7, 10)$

b) Find  $\text{SUBTRACT}(2, -4)$

c) Is  $\text{ADD}(2, \text{ADD}(x, -3)) = \text{ADD}(2x, -1)$ ?

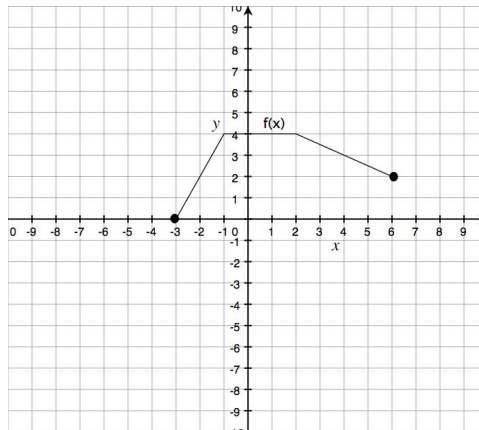
d) What is  $\text{SUBTRACT}(b, \text{ADD}(b, -b))$ ?

Algebra 2 Honors  
Final Exam 2015 – 2016 REVIEW

3. Given the graph of  $f(x)$ , graph  $g(x)$  and  $h(x)$ .

$$g(x) = f(x + 3) + 5$$

$$h(x) = -2f(x)$$



4. Complete the table.

Location of Point on the Original Function $y = f(x)$	Translated / Transformed Function	Type of translation / transformation (Check all that apply)	Location of Point on the Translated Function
(10, 10)	$y = f(x + 20) - 5$	<input type="checkbox"/> Vertical shift up by _____ <input type="checkbox"/> Vertical shift down by _____ <input type="checkbox"/> Horizontal shift left by _____ <input type="checkbox"/> Horizontal shift right by _____ <input type="checkbox"/> Vertical stretch by _____ <input type="checkbox"/> Vertical compression by _____ <input type="checkbox"/> Reflection over the x-axis	
(-5, 6)	$y = -3f(x) - 2$	<input type="checkbox"/> Vertical shift up by _____ <input type="checkbox"/> Vertical shift down by _____ <input type="checkbox"/> Horizontal shift left by _____ <input type="checkbox"/> Horizontal shift right by _____ <input type="checkbox"/> Vertical stretch by _____ <input type="checkbox"/> Vertical compression by _____ <input type="checkbox"/> Reflection over the x-axis	

5. Describe the translation of each graph from  $f(x)$  to  $g(x)$ .

$$f(x) = x^2 \longrightarrow g(x) = (x - 12)^2 + 6$$

$$f(x) = |x - 5| + 1 \longrightarrow g(x) = |x + 2| - 8$$

6. Evaluate and simplify each expression.

$$\text{Let } f(x) = |3x - 1| + 5, \quad g(x) = 4x + 1, \quad h(x) = \begin{cases} 6, & x < 5 \\ x^2 - 10, & x \geq 5 \end{cases}, \quad t(x) = x^2 - 2x$$

Algebra 2 Honors  
Final Exam 2015 – 2016 REVIEW

$$f(-10) =$$

$$h(-2) =$$

$$h(5) =$$

$$f(g(-1)) =$$

$$g\left(f\left(\frac{2}{3}\right)\right) =$$

$$g(3-b) =$$

$$f \circ g \circ h(0) =$$

$$g \circ t(x) =$$

$$t(g(x)) =$$

7. Evaluate and simplify each expression.

**f:**

<b>x</b>	5	-5	3	8	0	1	9
<b>y</b>	12	10	-1	3	0	-5	7

**g:**

<b>x</b>	-3	2	8	1	-4	7	6
<b>y</b>	0	-5	-5	8	1	3	5

$$f(5) =$$

$$g(1) =$$

$$f(-5) - g(2) =$$

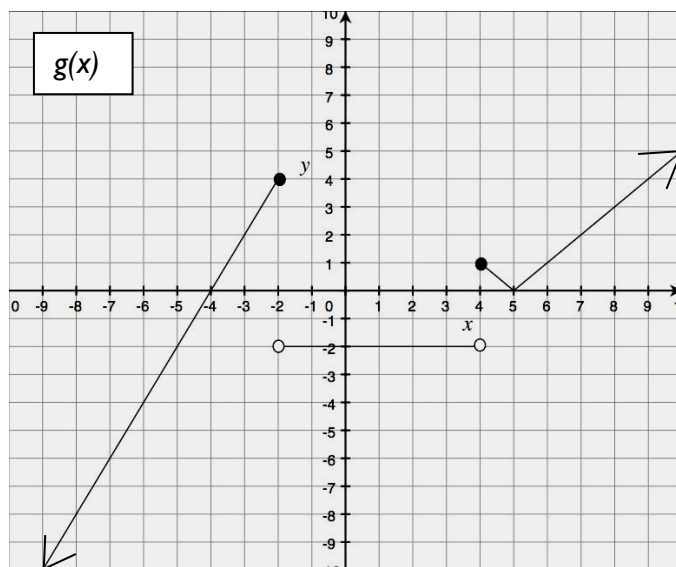
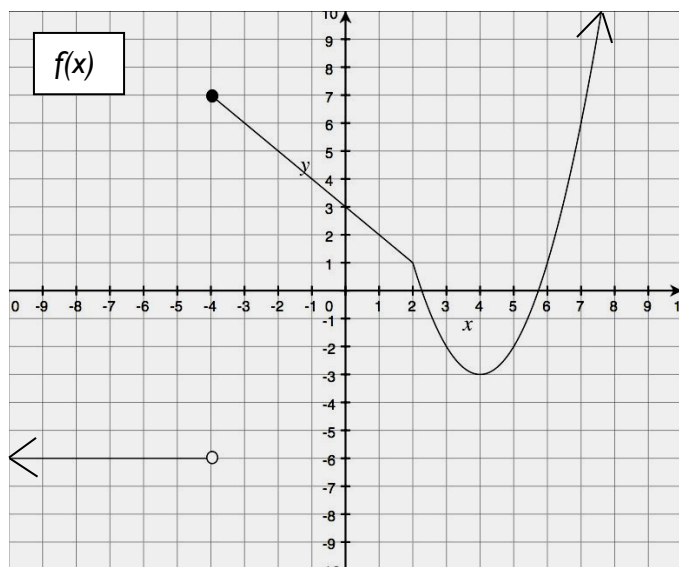
$$f(1)g(1) =$$

$$[f(9) + g(8)]^{f(8)} =$$

$$f(g(8)) =$$

$$g(f(9)) =$$

8. Evaluate each expression.



$$f(4) =$$

$$g(-2) =$$

$$f(-20) =$$

$$f(g(1.5)) =$$

$$g(f(4)) =$$

$$f(g(4)) =$$

$$(f + g)(7) =$$

$$f(-4) =$$

Algebra 2 Honors  
Final Exam 2015 – 2016 REVIEW

9.

Let  $f(x) = 3 - 2x$ ,  $g(x) = x^2 - 5x + 2$ ,  $h(x) = 3x - 5$ .

Find each of the following, and simplify.

1)  $(g + f)(x) =$

2)  $3f(x) - g(x) =$

3)  $(f \times g)(x) =$

4)  $(f \circ g)(x) =$

5)  $[h(x)]^2 =$

**Part 3: Graphical Analysis**

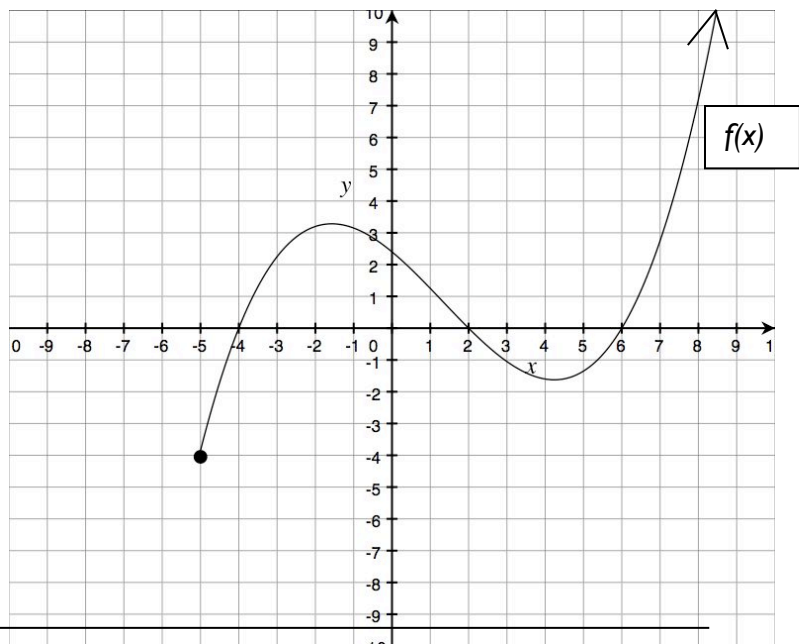
1) What is the domain of  $f(x)$ ?

2) What is the range of  $f(x)$ ?

3) For what values of  $x$  does  $f(x) = 0$ ?

4) What is the value of  $f(x)$  when  $x = 0$ ?

5) Over what intervals is  $f(x) \geq 0$ ?



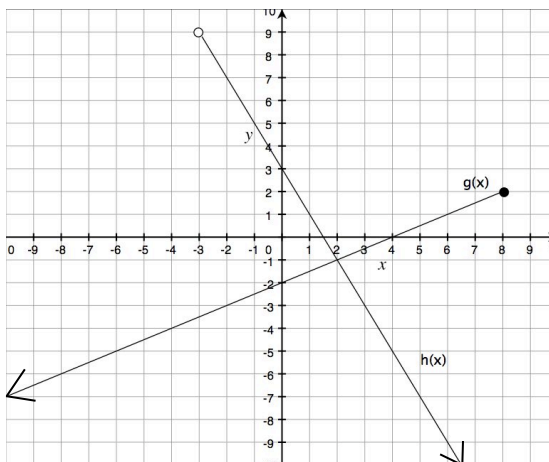
Algebra 2 Honors  
Final Exam 2015 – 2016 REVIEW

6) What is the domain of  $g(x)$ ?

7) What is the range of  $g(x)$ ?

8) What is the domain of  $h(x)$ ?

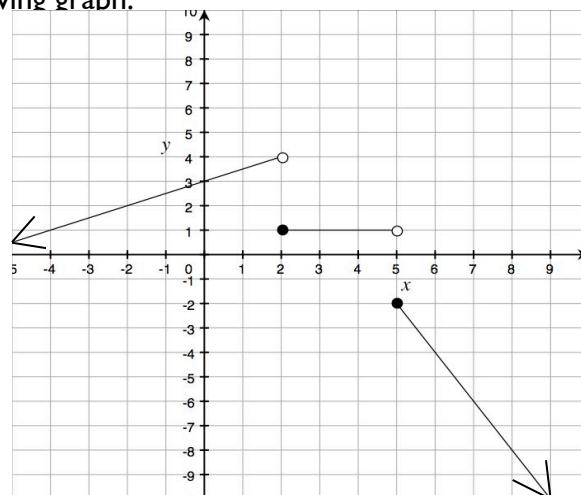
9) What is the range of  $h(x)$ ?



10)  $(h - g)(-2) =$     11)  $(h \times g)(2) =$     12)  $|h(5)| =$     13)  $(h \circ g)(8) =$

**Part 4: Piecewise Functions**

Write the piecewise function  $f(x)$  that would produce the following graph:



Graph  $f(x)$  on the coordinate plane. Then, find **exact** answers for the evaluation problems that follow.

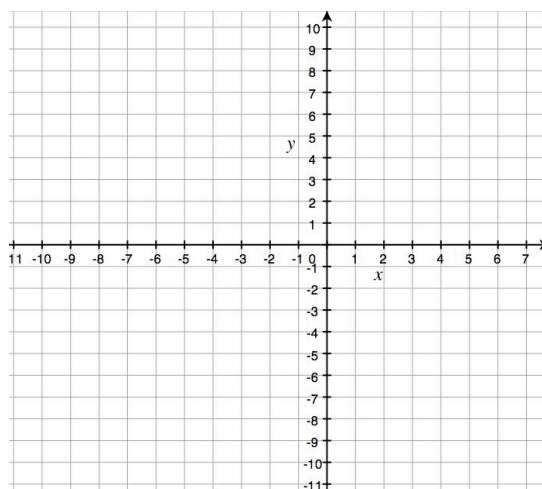
$$f(x) = \begin{cases} |x + 7| - 2, & -10 \leq x < -3 \\ 2, & -3 \leq x < 1 \\ -3x + 8, & x \geq 1 \end{cases}$$

$$f(-7) =$$

$$f(-2.65) =$$

$$f(1) =$$

$$f\left(\frac{5}{3}\right) =$$





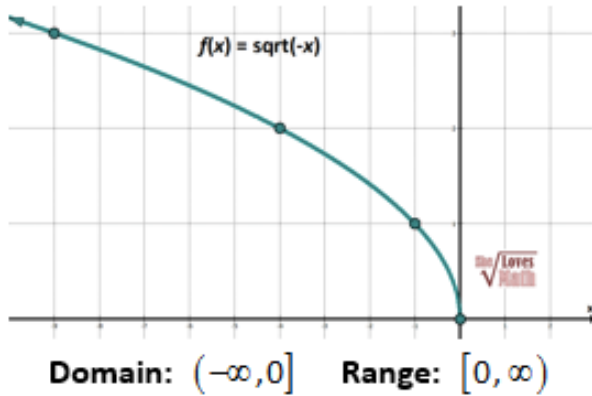
Algebra 2 Honors  
Final Exam 2015 – 2016 REVIEW

9. Determine whether the two functions are inverses. State how you know.

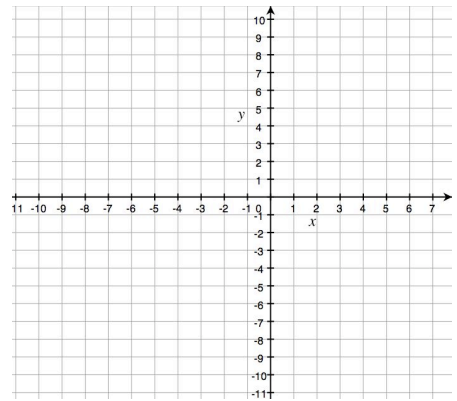
$$f(x) = x^2 - 5$$

$$g(x) = \sqrt{x} + 5$$

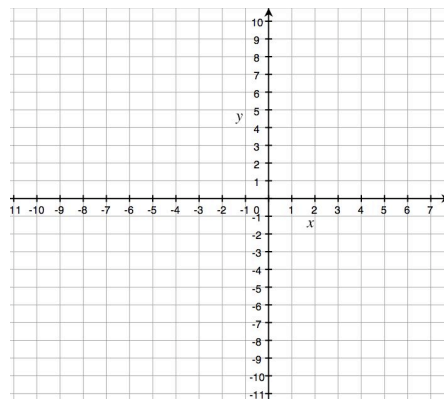
10. Use the following graph to answer the questions.



a) Draw  $f(x)$  rotated  $180^\circ$  about the origin, and state the equation of the transformed function.



b) Draw  $f(x)$  reflected across the y-axis, and state the equation of the transformed function.

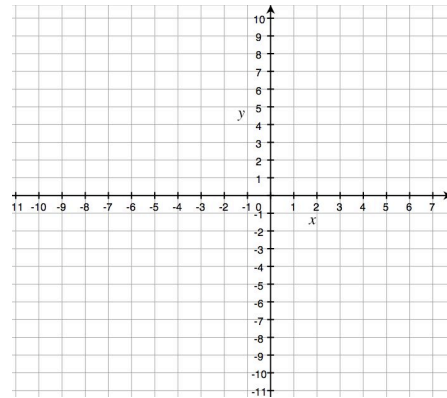


Algebra 2 Honors  
Final Exam 2015 – 2016 REVIEW

c) Draw  $f(x)$  reflected across the x-axis, and state the equation of the transformed function

3)

4)



**SYSTEMS OF EQUATIONS WITH FUNCTIONS:**

1) Functions  $f$  and  $g$  are defined below.

$$\begin{cases} f(x) = \frac{1}{4x} \\ g(x) = x^2 \end{cases}$$

The graphs of  $y = f(x)$  and  $y = g(x)$  intersect at point  $P$ . Determine the  $x$ -coordinate of  $P$ . Leave your answer in simplest radical form.

2) Find the solution to this system of equations:

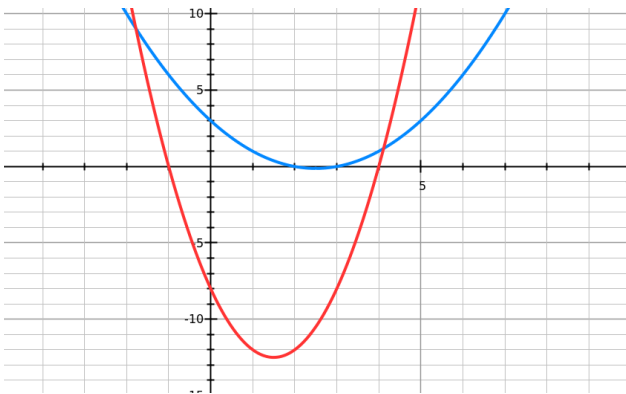
$$-5x + y = 12$$

$$-3(x - 2y) = 4$$

3)

**Directions:**

1. Write the equation of each parabola in **each** form (**standard, vertex, and factored**).
2. Identify the **vertex** and **zeroes/x-intercepts** of each function.
3. Find the points of intersection. Leave all irrational numbers in simplest radical form.



Algebra 2 Honors  
Final Exam 2015 – 2016 REVIEW

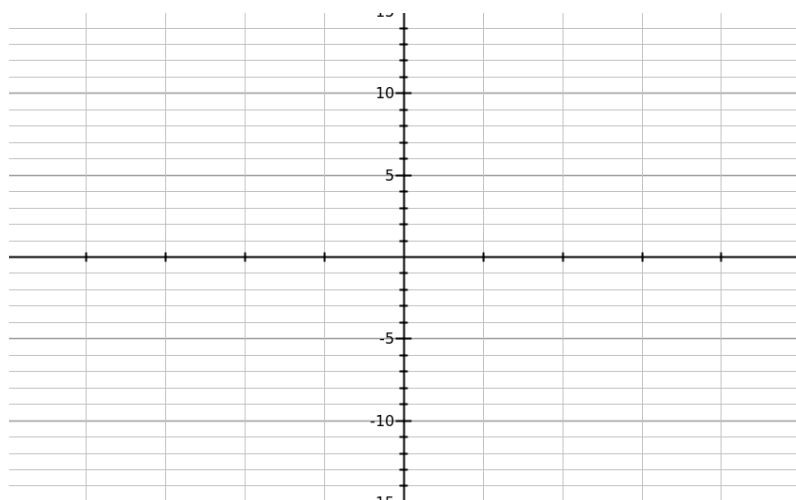
**Directions:**

1. Draw the graphs of each of the following three functions on the graph below.
2. Identify the **vertex** and **zeroes/x-intercepts** of each function.
3. Find the points of intersection. Leave all irrational numbers in simplest radical form.

$$h(x) = (2x - 5)(x - 2)$$

$$k(x) = -x^2 - \frac{1}{2}x + 5$$

$$m(x) = 2\left(x + \frac{1}{4}\right)^2 - 10$$



4)